Geometry: 9.1-9.3 Notes

NAME______________________

9.1 The Pythagorean Theorem__________ ______________________________ **Date:**________________

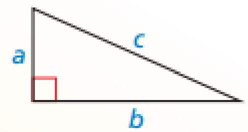
Define Vocabulary:

Pythagorean triple

Theorem 9.1 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Proof Explorations 1 and 2, p. 463; Ex. 39, p. 484



$c^2 = a^2 + b^2$

A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

Common Pythagorean Triples and Some of Their Multiples

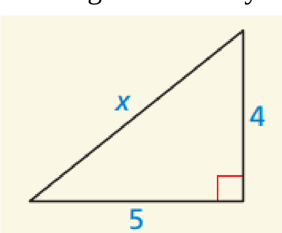
3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold-faced triple by the same factor.

Examples: Using the Pythagorean Theorem

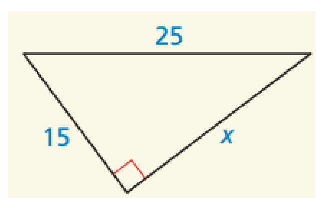
<u>WE DO</u>

Find the value of x. Then tell whether the side lengths form a Pythagorean triple.



<u>YOU DO</u>

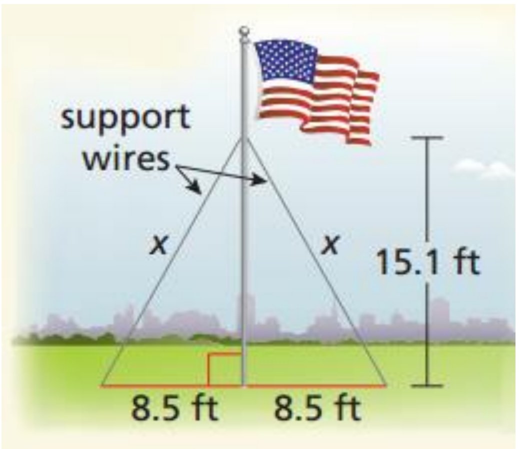
Find the value of x. Then tell whether the side lengths form a Pythagorean triple.



Examples: Solving a Real-Life Problem

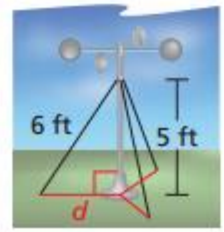
WE DO

The flagpole shown is supported by two wires. Use the Pythagorean Theorem to approximate the length of each wire.



YOU DO

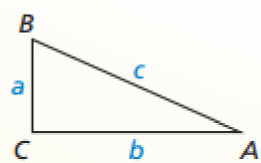
An anemometer is a device used to measure wind speed. The anemometer shown is attached to the top of a pole. Support wires are attached to the pole 5 feet above the ground. Each support wire is 6 feet long. How far from the base of the pole is each wire attached to the ground?



Theorem 9.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

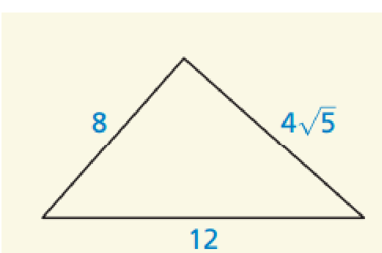


Proof Ex. 39, p. 470

Examples: Verifying Right Triangles

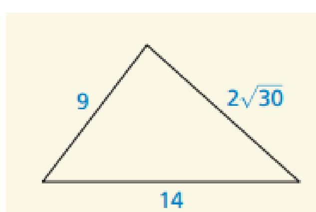
WE DO

Tell whether each triangle is a right triangle.



YOU DO

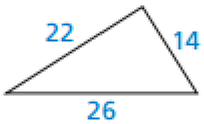
Tell whether each triangle is a right triangle.



Examples: Verifying Right Triangles

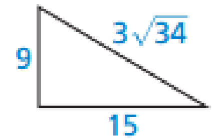
WE DO

Tell whether each triangle is a right triangle.



YOU DO

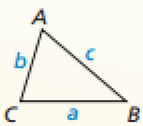
Tell whether each triangle is a right triangle.



Theorem 9.3 Pythagorean Inequalities Theorem

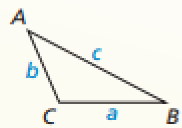
For any $\triangle ABC$, where c is the length of the longest side, the following statements are true.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



$c^2 < a^2 + b^2$

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



$c^2 > a^2 + b^2$

Proof Exs. 42 and 43, p. 470

Examples: Classifying Triangles

WE DO

Verify that segments with lengths of 2.1, 2.8, and 3.5 form a triangle. Is the triangle acute, right, or obtuse?

YOU DO

Verify that segments with lengths of 3, 4, and 6 form a triangle. Is the triangle acute, right, or obtuse?

Verify that segments with lengths of 14 meters, 15 meters, and 11 meters form a triangle. Is the triangle acute, right, or obtuse?

Verify that segments with lengths of 90, 216, and 234 form a triangle. Is the triangle acute, right, or obtuse?

Assignment	

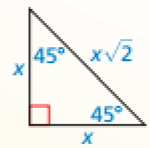
9.2 Special Right Triangles______________________ ______________________________ Date:______________

Define Vocabulary:

isosceles triangle

Theorem 9.4 45°- 45°- 90° Triangle Theorem

In a 45°- 45°- 90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



hypotenuse = leg $\cdot \sqrt{2}$

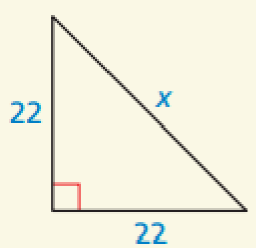
Proof Ex. 19, p. 476

Examples: Finding Side Lengths in 45°- 45°- 90° Triangles

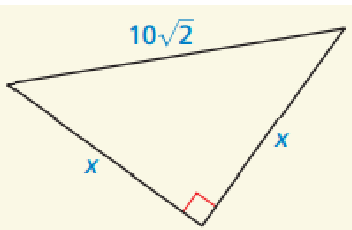
WE DO

Find the value of x. Write your answer in simplest radical form.

a.



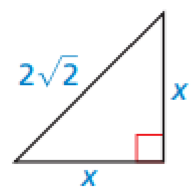
b.



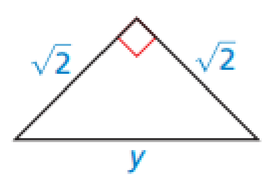
YOU DO

Find the value of the variable. Write your answer in simplest radical form.

1.

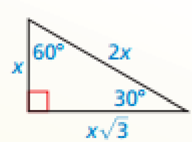


2.



Theorem 9.5 30°- 60°- 90° Triangle Theorem

In a 30°- 60°- 90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



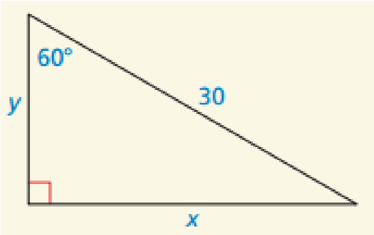
hypotenuse = shorter leg $\cdot$ 2
longer leg = shorter leg $\cdot \sqrt{3}$

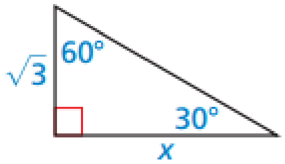
Proof Ex. 21, p. 476

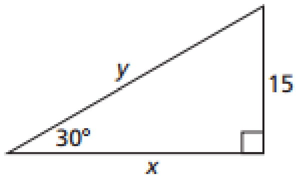
Examples: Finding Side Lengths in a 30°- 60°- 90° Triangle

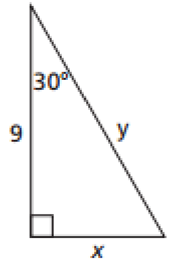
WE DO | **YOU DO**

Find the value of the variable(s). Write your answer in simplest radical form.









Examples: Modeling with mathematics

WE DO

A warning sticker is shaped like an equilateral triangle with side length of 4 inches. Estimate the area of the sticker by finding the area of the equilateral triangle to the nearest tenth of an inch.

YOU DO

Sketch the figure that is described. Find the indicated length. Round decimal answers to the nearest tenth. An isosceles triangle with 30° base angles has an altitude of 3 meters. Find the length of the base of the isosceles triangle.

Assignment	

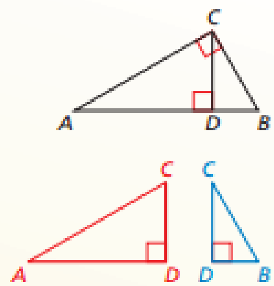
9.3 Similar Right Triangles____Date:____________

Define Vocabulary:

geometric mean

Theorem 9.6 Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



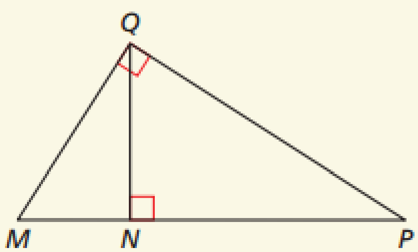
$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

Proof Ex. 45, p. 484

Examples: Identifying Similar Triangles.

WE DO

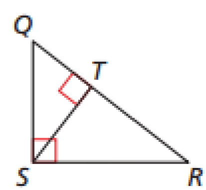
Identify the similar triangles in the diagram.



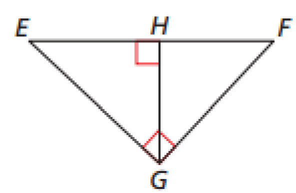
YOU DO

Identify the similar triangles.

1.



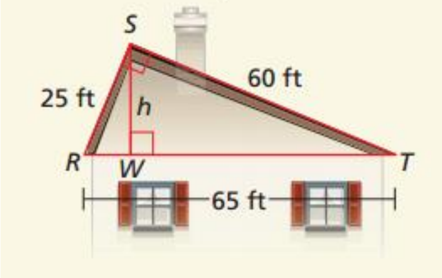
2.



Examples: Modeling with Mathematics

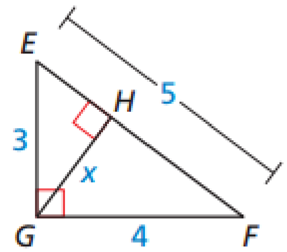
WE DO

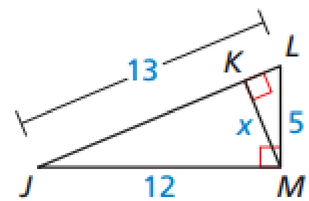
A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height h of the roof.



YOU DO

Find the value of x.





Geometric Mean

The **geometric mean** of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

Examples: Finding a geometric mean

WE DO

Find the geometric mean of 8 and 10.

Find the geometric mean of 12 and 27.

YOU DO

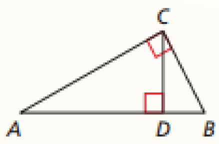
Find the geometric mean of 18 and 54.

Find the geometric mean of 16 and 18.

Theorem 9.7 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.



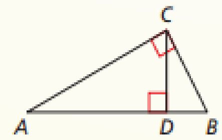
$CD^2 = AD \cdot BD$

Proof Ex. 41, p. 484

Theorem 9.8 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



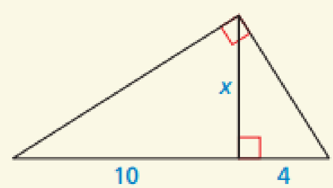
$CB^2 = DB \cdot AB$

$AC^2 = AD \cdot AB$

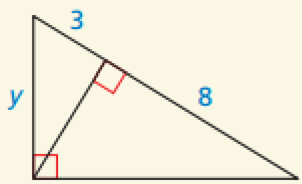
Examples: Find the value of each variable using geometric mean.

<u>WE DO</u>

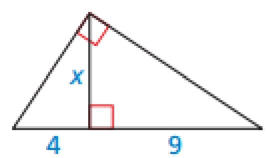
a.



b.



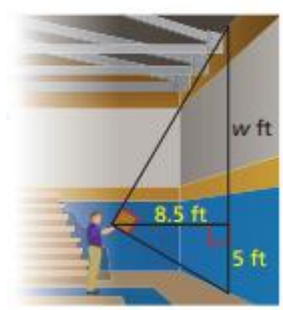
<u>YOU DO</u>



Examples: Using Indirect Measurement.

<u>WE DO</u>

Use the diagram and information in Example 5 from the book. The vertical distance from the ground to your eye is 5.4 feet and the distance from you to the gym wall is 8.1 feet. Approximate the height of the gym wall.



Assignment	